

Magnetic coupling between mesoscopic superconducting rings

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(February 1, 2008)

Using the nonlinear Ginzburg-Landau theory we investigated the dependence of the magnetic coupling between two concentric mesoscopic superconducting rings on their thickness. The size of this magnetic coupling increases with the thickness of the rings.

I. INTRODUCTION

Single mesoscopic superconducting samples have been studied extensively during the last years. The main part of the studies covered single superconducting disks (see for example Refs. [1-3]) and rings (see for example Refs. [4-7]). The properties of single mesoscopic rings of finite thickness are studied in detail in our previous papers [5]. A single mesoscopic superconducting ring with a finite thickness tries to expell the applied magnetic field by inducing supercurrents which create a local magnetic field opposite to the external one. In some regions the field will be expelled to the outside, in other regions it will be compressed into the hole of the ring. As a consequence the total magnetic field is strongly nonuniform in the neighbourhood of a single superconductor.

What happens if another superconducting ring is placed in the center of such a ring? The inner ring will try to expell the total nonuniform field instead of the externally applied one. And this expulsion will influence the outer ring. In this way, both rings are coupled through the magnetic field. This coupling will influence the properties of both rings.

Recently, Morelle et al. [8] studied experimentally the interaction between two concentric superconducting aluminium rings, close to the superconducting/normal transition. Due to the coupling between the two rings, they found changes in the $T_c(H)$ oscillations of the outer ring.

In the present paper we will investigate theoretically the influence of the sample thickness on the coupling between two superconducting mesoscopic rings. With increasing sample thickness the expulsion of the magnetic field will be more complete, which means that the total field becomes more nonuniform and the coupling between both rings increases.

II. THEORETICAL FORMALISM

We consider two concentric mesoscopic superconducting rings made of the same material and with the same thickness d , immersed in an insulating medium and placed

in a perpendicular magnetic field H_0 . The smaller ring has inner (outer) radius R_i^* (R_o^*) and the larger ring inner (outer) radius R_i (R_o). To solve this problem, we expand the approach for thin superconducting disks of Ref. [2] to a system of two axial symmetric superconductors. We solve the two coupled Ginzburg-Landau equations self-consistently

$$\left(-i\vec{\nabla} - \vec{A}\right)^2 \Psi = \Psi \left(1 - |\Psi|^2\right), \quad (1)$$

$$-\kappa^2 \Delta \vec{A} = \frac{1}{2i} \left(\Psi^* \vec{\nabla} \Psi - \Psi \vec{\nabla} \Psi^*\right) - |\Psi|^2 \vec{A}, \quad (2)$$

where we express all distances in units of the coherence length ξ , the order parameter in $\Psi_0 = \sqrt{-\alpha/\beta}$ and the vector potential in $c\hbar/2e\xi$.

The boundary condition for the order parameter is

$$\vec{n} \cdot \left(-i\vec{\nabla} - \vec{A}\right) \Psi \Big|_{\rho=R_i^*, R_o^*, R_i, R_o} = 0. \quad (3)$$

The boundary condition for the vector potential has to be taken far away from the sample, where the field equals the applied magnetic field.

The difference between the Gibbs free energy of the superconducting state and the normal state is determined by the expression

$$F = \frac{1}{V} \int \left[2 \left(\vec{A} - \vec{A}_0 \right) \cdot \vec{j} - |\Psi|^4 \right] d\vec{r}, \quad (4)$$

where the integration is over the total volume V of the superconducting samples and \vec{A}_0 is the vector potential corresponding to the applied uniform field.

Since we consider sufficiently narrow rings only giant vortex states will nucleate, which can be characterized by their angular momentum or vorticity. Therefore, the superconducting states in a double ring system can be denoted as (L_{out}, L_{in}) , with L_{out} (L_{in}) the vorticity of the outer (inner) ring.

III. RESULTS

As an example, we consider a superconducting ring with radii $R_o = 2.0\xi$ and $R_i = 1.5\xi$ with a smaller ring in the center with radii $R_o = 2.0\xi$ and $R_i^* = 0.6\xi$. Both rings have thus the same width and we assume that they have the same thickness d and are made of the same material with $\kappa = 0.28$.

Fig. 1 shows the free energy (solid curves) as a function of the applied magnetic field H_0 for a double ring configuration with thickness $d = 0.5\xi$. The free energies of the single rings with the same size as the inner and the outer ring are given by the thick dashed and the thick dotted curves, respectively. Notice that everywhere in this paper, the free energy is expressed in units of $F_0 = H_{c2}V/8\pi$, where V is the sum of the volumes of the two rings. This is the reason why the free energy of a single ring is not equal to $-F_0$ at zero magnetic field as it was in, for example, Ref. [5]. In the single inner ring, superconducting states can nucleate with vorticities $L_{in} = 0, 1$ and 2 , and in the single outer ring with $L_{out} = 0$ up to $L_{out} = 10$. The superconducting/normal transition fields are $H_{c3} \approx 6.5H_{c2}$ for the inner ring and $H_{c3} \approx 6.75H_{c2}$ for the outer ring. The indices (L_{out}, L_{in}) in Fig. 1 indicate the ground states of the double ring configuration. Notice that, as compared to the single outer ring, an extra ground state transition occurs at $H_0/H_{c2} \approx 1.5H_{c2}$, where the vorticity of the outer ring stays the same, $L_{out} = 2$, and the vorticity of the inner ring changes with one unit, $L_{in} = 0 \rightarrow 1$. Hence, by putting an extra ring in the center of a larger ring, the ground state shows extra transitions. This result corresponds to the experimental result of Morelle et al. [8]. Notice further that at $H_0/H_{c2} \approx 4.3H_{c2}$ both vorticities change with one unit. The ground state changes from the (6,1) state to the (7,2) state.

Next we investigate the strength of the interaction between the two rings. In Fig. 2 we plot the ground state free energy of the coupled rings (solid curves) and the sum of the ground state free energies of the two non-interacting single rings (dashed curves) as a function of the applied magnetic field for three values of the sample thickness: $d/\xi = 0.15, 0.5$, and 1.0 . The insets show some crossings in more detail. The difference between both curves is the interaction energy between the two rings. The middle curves correspond to the configuration of Fig. 1, i.e. $d/\xi = 0.5$. The interaction is most pronounced for the (2,0), the (5,1) and the (6,1) state, i.e. at fields just below the transition fields of the inner ring. Due to this interaction, not only the value of the free energy differs, but also the transition fields change. For example, the $(1,0) \rightarrow (2,0)$ transition occurs at $H_0/H_{c2} \approx 1.01H_{c2}$ neglecting the coupling and at $H_0/H_{c2} \approx 1.07H_{c2}$ including the coupling. Moreover, the right inset shows clearly that, neglecting the interaction, the ground state changes from the (6,1) state into the (6,2) state and then into the (7,1) state, while including the interaction it transits directly from the (6,1) state into the (7,2) state. Thus, the coupling between the two rings leads to the interesting result that the (6,2) state is no longer a ground state.

It is known that the expulsion of the field from the superconducting rings increases with increasing sample thickness. This is also shown in Fig. 3, where we plot the radial distribution of the magnetic field for the (5,1) state of the double ring configuration of Fig. 1 at $H_0/H_{c2} =$

3.52 for three values of the thickness, i.e. $d = 0.15\xi, 0.5\xi$, and 1.0ξ . Since the interaction energy is due to the magnetic coupling between both rings, its value depends strongly on the thickness of the system. This can be seen from Fig. 2. As compared to the above considered case ($d/\xi = 0.5$) the interaction between the two rings is much smaller for $d/\xi = 0.15$, i.e. the ground state free energy including the coupling is much closer to the one neglecting the coupling. On the other hand, for $d/\xi = 1.0$ the interaction energy is larger than for $d/\xi = 0.5$. Also the number of ground state transitions of the coupled ring configuration depends on the sample thickness. For $d/\xi = 0.15$ the (6,2) state is a ground state in the region $4.27 \leq H_0/H_{c2} \leq 4.28$, while for $d/\xi = 0.5$ and $d/\xi = 1.0$ the ground state changes from the (6,1) state directly to the (7,2) state.

IV. CONCLUSIONS

We investigated the dependence of the magnetic coupling between two concentric mesoscopic superconducting rings on the sample thickness. This coupling results in: 1) the free energy of the double ring configuration is not exactly the same as the sum of the free energies of the two single rings. The difference between the two energies, the interaction energy, increases with increasing sample thickness. 2) For sufficiently thick samples, some superconducting states are no longer realized as ground states.

Acknowledgement 1

This work was supported by the Flemish Science Foundation (FWO-VI), the "Onderzoeksraad van de Universiteit Antwerpen", the "Interuniversity Poles of Attraction Program - Belgian State, Prime Minister's Office - Federal Office for Scientific, Technical and Cultural Affairs", and the European ESF-Vortex Matter. Discussions with V. V. Moshchalkov are gratefully acknowledged.

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FIG. 1. The free energy as a function of the applied magnetic field for a superconducting ring with radii $R_i^* = 0.6\xi$ and $R_o^* = 1.1\xi$ (thick dotted curves) and for a ring with radii $R_i = 1.5\xi$ and $R_o = 2.0\xi$ (thick dashed curves) and the double ring configuration (thin solid curves).

FIG. 2. The ground state free energy of the double ring configuration of Fig. 1 (solid curves) and the sum of the ground state free energies of both single rings (dashed curves) for three values of the sample thickness: $d/\xi = 0.15$ (upper curves, shifted over $0.2F_0$), $d/\xi = 0.5$ (middle curves, shifted over $0.1F_0$) and $d/\xi = 1.0$ (lower curves) The insets show some crossings in more detail.

FIG. 3. The radial distribution of the magnetic field for the (5,1) state of the double ring of Fig. 1 at $H_0/H_{c2} = 3.52$ for three values of the sample thickness: $d/\xi = 0.15$ (solid curves), $d/\xi = 0.5$ (dashed curves) and $d/\xi = 1.0$ (dotted curves).





